

Cosmology with Hu-Sawicki gravity in Palatini Formalism

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Cosmological models based on $f(R)$ -gravity may exhibit a natural acceleration mechanism without introducing a dark energy component. In this paper, we investigate cosmological consequences of the so-called Hu-Sawicki $f(R)$ gravity in the Palatini formalism. We derive theoretical constraints on the model parameters and perform a statistical analysis to determine the parametric space allowed by current observational data. We find that this class of models is indistinguishable from the standard Λ CDM model at the background level. Differently, from previous results in the metric approach, we show that these scenarios are able to produce the sequence of radiation-dominated, matter-dominated, and accelerating periods without need of dark energy.

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I. INTRODUCTION

General Relativity (GR) is a very well-tested and established theory of gravity. However, all the successful tests performed so far do not account for the ultra-large scales corresponding to the low curvature characteristics of the Hubble radius today. Thus, in principle it is conceivable that the recent discovery of the late-time cosmic acceleration [1] could be explained by modifying Einstein's GR in the far infrared regime. Although the diversity of approaches is the hallmark in this field (see, e.g., Refs. [2]), the simplest possible theory results by adding terms proportional to powers of the Ricci scalar R to the Einstein-Hilbert Lagrangian, the so-called $f(R)$ gravity (see Refs. [3] for recent reviews).

Differently from the standard general relativistic scenario, $f(R)$ cosmology can naturally drive an accelerating cosmic expansion without introducing dark energy. However, the freedom in the choice of different functional forms of $f(R)$ gives rise to the problem of how to constrain the many possible $f(R)$ gravity theories. In this regard, much efforts have been developed so far, mainly from the theoretical viewpoint [4]. General principles such as the so-called energy conditions [5], non-local causal structure [6], and issues such as loop quantum cosmology [7], have also been taken into account in order to clarify its subtleties. More recently, observational constraints from several cosmological data sets have also been explored for testing the viability of these theories [8].

An important aspect that is worth emphasizing concerns the two different variational approaches that may be followed when one works with $f(R)$ gravity, namely, the metric and the Palatini formalisms. In the metric

formalism the connections are defined *a priori* as the Christoffel symbols of the metric and the variation of the action is taken with respect to the metric, whereas in the Palatini variational approach the metric and the affine connections are treated as independent fields and the variation is taken with respect to both (for a review on $f(R)$ theories in the Palatini approach see [9]). The result is that in the Palatini approach the connections depend on the metric and also on the particular $f(R)$, while in the metric formalism the connections depend only on the metric. We have then that the same $f(R)$ leads to different spacetime structures.

These differences also extend to the observational aspects. For instance, we note that some cosmological models based on a power-law functional form in the metric formulation fail in reproducing the standard matter-dominated era followed by an acceleration phase [10] (see, however, [11]), whereas in the Palatini approach, some analysis have shown that such theories admit the three post inflationary phases of the standard cosmological model [12]. Nevertheless, we do not have yet a clear comprehension of the properties of the Palatini formulation of $f(R)$ gravity in other scenarios, and issues such as the Newtonian limit [13] and the Cauchy problem [14] are still contentious (see, however, Refs. [15]). Another issue with the Palatini $f(R)$ formulation concerns surface singularities of static spherically symmetric objects with polytropic equation of state [9, 16]. This problem has been reexamined in Refs. [17], where the authors showed that the singularities may have more to do with peculiarities of the polytropic equation of state used (e.g. its natural regime of validity) and the $f(R)$ model chosen.

Although being mathematically simpler than the metric formulation, the Palatini approach has received little attention from the point of view of cosmological tests. Following early works in this direction [12, 18, 19], in this paper we explore the cosmological consequences of a class of modified $f(R)$ gravity, recently proposed by Hu & Sawicki [20], in the Palatini formulation. Among a number of $f(R)$ models discussed in the literature, this is designed to possess a chameleon mechanism that allows

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to evade solar system constraints and the cosmological scenario that arises from the metric formalism of this model has been shown to satisfy the conditions needed to produce a cosmologically viable expansion history. In order to test the observational viability of this class of models in the Palatini approach, we use different kinds of observational data, namely: type Ia supernovae (SNe Ia) observations from Union2.1 sample [21], estimates of the expansion rate at $z \neq 0$, as discussed in Ref. [22], current measurements of the product of the CMB acoustic scale ℓ_A and the comoving sound horizon at photon decoupling (CMB/BAO ratio) [23] and measurements of the gas mass fraction in galaxy clusters [24]. We show that for a subsample of model parameters, the so-called Hu-Sawicki scenarios in the Palatini approach are indistinguishable from the standard Λ CDM model, being able to produce the sequence of radiation-dominated, matter-dominated, and accelerating periods without need of dark energy.

II. THE PALATINI APPROACH

The modified Einstein-Hilbert action that defines an $f(R)$ gravity is given by

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m, \quad (1)$$

where $\kappa^2 = 8\pi G$, g is the determinant of the metric tensor and S_m is the standard action for the matter fields.

In the Palatini variational approach the fundamental idea is to regard the torsion-free connection $\Gamma_{\mu\nu}^\rho$, as well as the metric $g_{\mu\nu}$ entering the action above, as two independent fields. The field equations in this approach are

$$f_R R_{(\mu\nu)} - \frac{f}{2} g_{\mu\nu} = \kappa^2 T_{\mu\nu} \quad (2a)$$

and

$$\tilde{\nabla}_\beta (\sqrt{-g} f_R g^{\mu\nu}) = 0, \quad (2b)$$

where $f_R = df/dR$ and $\tilde{\nabla}$ represents covariant derivatives. From the above equations, we obtain the connections

$$\Gamma_{\mu\nu}^\rho = \{\rho_{\mu\nu}\} + \frac{1}{2f_R} (\delta_\mu^\rho \partial_\nu + \delta_\nu^\rho \partial_\mu - g_{\mu\nu} g^{\rho\sigma} \partial_\sigma) f_R, \quad (3)$$

where $\{\rho_{\mu\nu}\}$ are the Christoffel symbols of the metric. As mentioned earlier, the connections, and so the gravitational fields, are described not only by the metric $g_{\mu\nu}$ but also by the proposed $f(R)$ theory. As a cosmological model we consider a homogeneous isotropic Universe described by the Friedmann-Lemaître-Robertson-Walker (FLRW) flat geometry $g_{\mu\nu} = \text{diag}(-1, a^2, a^2, a^2)$, where $a(t)$ is the cosmological scale factor, and as the source of curvature a perfect-fluid with energy density ρ and

pressure p whose energy-momentum tensor is given by $T_\nu^\mu = \text{diag}(-\rho, p, p, p)$.

The generalized Friedmann equation, obtained from (2) and (3), can be written in terms of the redshift z as

$$\frac{H^2}{H_0^2} = \frac{3\Omega_{m0}(1+z)^3 + 6\Omega_{r0}(1+z)^4 + f(R)/H_0^2}{6f_R \xi^2}, \quad (4)$$

where

$$\xi = 1 + \frac{9}{2} \frac{f_{RR}}{f_R} \frac{H_0^2 \Omega_{m0} (1+z)^3}{R f_{RR} - f_R}, \quad (5)$$

and Ω_{m0} and Ω_{r0} stand for the present-day values of the matter and radiation density parameters. The trace of (2a) results in

$$R f_R - 2f = -3H_0^2 \Omega_{m0} (1+z)^3, \quad (6)$$

which we will consider in order to restrict the parameters of the given $f(R)$ theory.

A. The Hu-Sawicki gravity model

In this paper we are interested in the $f(R)$ modified gravity model proposed by Hu and Sawicki [20]

$$f(R) = R - m^2 \frac{c_1 \left(\frac{R}{m^2}\right)^n}{c_2 \left(\frac{R}{m^2}\right)^n + 1}. \quad (7)$$

For the case $n = 1$, the above equation can be rewritten as [26]

$$f(R) = R - \frac{2\Lambda}{\frac{\mu^2}{R} + 1},$$

where $\Lambda = m^2 c_1 / 2c_2$ and $\mu^2 = m^2 / c_2$ are dimensionless parameters and $m^2 = H_0^2 \Omega_{m0}$. Note that, in the regime $R \gg \mu^2$, the model above is practically indistinguishable from the Λ CDM scenario¹. We have verified that the parameter n is completely unconstrained by current data. Thus, without loss of generality and following Ref. [26], we focus our analyses on the case $n = 1$.

The model above is intended to explain the current acceleration of the universe without considering a cosmological constant term. Cosmological and astrophysical constraints on the Hu-Sawicki gravity scenario in the metric formalism have been examined in a number of papers (see, e.g., Refs. [27]). However, there is a complete lack of studies on the mechanism of this proposal in the Palatini approach. Note that, irrespective of the formalism adopted, the model (7) must satisfy certain viability

¹ As shown in Ref. [26], this is equivalent to have $|f_{R0}| \sim 1$, where the subscript 0 denotes present-day quantities. See also Table I for some estimates of this quantity.

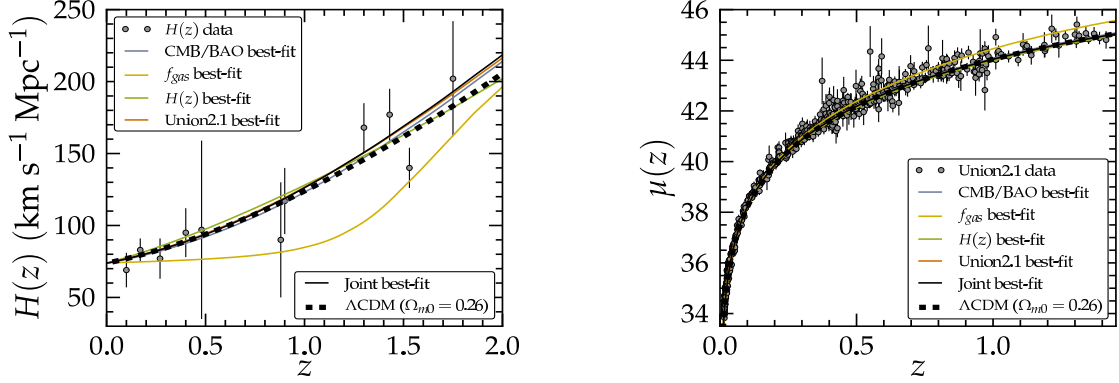


FIG. 1: (*left*) The predicted Hubble evolution $H(z)$ as a function of the redshift for the Hu-Sawicki model in the Palatini formalism (Eqs. 4-7). The curves correspond to the best-fit values of Ω_{m0} and c_2 discussed in the text. For the sake of comparison, the standard Λ CDM model prediction is also shown. The data points are the measurements of the $H(z)$ given in Ref. [22]. (*right*) Hubble diagram for 580 SNe Ia from the Union2.1 sample [21]. The curves correspond to the best-fit values of Ω_{m0} and c_2 displayed in Table I.

conditions, as for example, the positivity of the effective gravitational coupling which requires $\kappa^2/f_R > 0$ (to avoid anti-gravity). For the current epoch, this condition implies

$$f_{R0} = \frac{3 + 2(\frac{R_0}{m^2})^2 c_2}{\frac{R_0}{m^2} [1 + 2(\frac{R_0}{m^2}) c_2]} > 0, \quad (8)$$

which corresponds to the following bounds for the parameter c_2 :

$$c_2 < -\frac{1}{2} \frac{m^2}{R_0} \quad \text{or} \quad c_2 > -\frac{3}{2} \left(\frac{m^2}{R_0} \right)^2. \quad (9)$$

III. OBSERVATIONAL CONSTRAINTS

In order to perform the observational tests discussed below, we solve Eq. (4) by taking the following steps: i) we first set $z = 0$ and compute c_1 (in terms of c_2 and R_0) from Eq. (6); ii) we combine the latter result with Eq. (4) in order to obtain a solution for R_0 ; iii) the value of R_0 is then used as initial condition to solve Eq. (6) by using a fourth-order Runge-Kutta method and, finally, to obtain the function $H(z)$, as given by (4).

Fig. 1 shows the evolution of the Hubble parameter and the predicted distance modulus $\mu(z) = 5 \log[d_L(z)/\text{Mpc}] + 25$, where $d_L = (1+z) \int_0^z \frac{dz'}{H(z')}$ stands for the luminosity distance, as a function of redshift for some best-fit values for Ω_{m0} and c_2 obtained in our analyses (see Sec. IIIC). For the sake of comparison, the standard Λ CDM prediction with $\Omega_{m0} = 0.26$ is also shown (dashed line).

A. Expansion rate

In order to test the observational viability of the $f(R)$ scenario described by Eq. (7), we firstly use current measurements of the expansion rate at $z \neq 0$. Currently, this test is based on the fact that Luminous Red Galaxies (LRGs) can provide us with direct measurements of $H(z)$ [28] (see also [29] for a recent review on $H(z)$ measurements from different techniques). This can be done by calculating the derivative of cosmic time with respect to redshift, i.e., $H(z) \simeq -\frac{1}{(1+z)} \frac{\Delta z}{\Delta t}$. This method was first presented in [28] and consists in measuring the age difference between two red galaxies at different redshifts in order to obtain the rate $\Delta z/\Delta t$. By using a recent released sample from Gemini Deep Survey (GDDS) [30] and archival data [31], Ref. [22] has calculated 11 $H(z)$ data points over the redshift range $0.1 \leq z \leq 1.75$.

We estimate the free parameters \mathbf{P} by using a χ^2 statistics, i.e.,

$$\chi_H^2 = \sum_{i=1}^{N_H} \frac{[H_{th}^i(z_i|\mathbf{P}) - H_{obs}^i(z_i)]^2}{\sigma_i^2}, \quad (10)$$

where $H_{th}^i(z|\mathbf{P})$ is the theoretical Hubble parameter at redshift z_i and σ_i is the uncertainty for each of the $N_H = 11$ determinations of $H(z)$ given in [22]. In our analyses, the Hubble constant H_0 is considered as a nuisance parameter and we marginalize over it.

B. SNe Ia

The predicted distance modulus for a supernova at redshift z , given a set of parameters \mathbf{P} , is $\mu_i^p(z_i|\mathbf{P}) = 5 \log D_L(z_i, \mathbf{P}) + \mu_0$, where $D_L = H_0 d_L$ is the Hubble-free

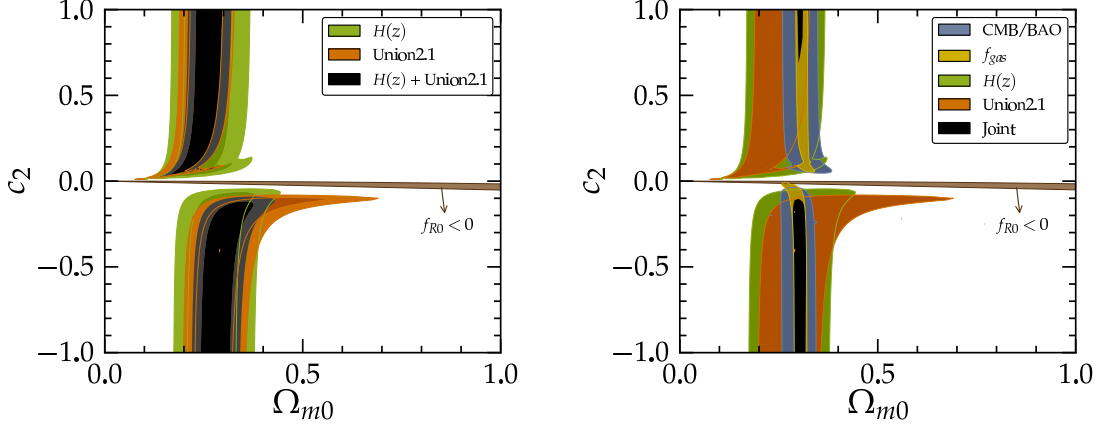


FIG. 2: (left) Contours of $\Delta\chi^2 = 2.3$ and $\Delta\chi^2 = 6.17$ in the $\Omega_{m0} \times c_2$ plane arising from SNe Ia union2.1 sample [21] (Orange) and current $H(z)$ data [22] (Green). Black contours stand for the joint analysis involving these two sets of data. (right) Contours of $\Delta\chi^2 = 6.17$ in the $\Omega_{m0} \times c_2$ plane when 7 measurements of the CMB/BAO ratio and 57 measurements of the gas mass fraction of galaxy clusters are added. We have also indicated the region $f_{R0} < 0$ corresponding to the bounds (9) for the best-fit values shown in Table I.

TABLE I: Best-fit values for c_2 , c_1 and f_{R0} . The error bars for Ω_{m0} correspond to 68.3% and 95.4% confidence intervals.

Test	Ω_{m0}	c_2	c_1	f_{R0}	χ^2_{min}	χ^2_r
$H(z)$	0.22 $^{+0.11}_{-0.08}$ $^{+0.19}_{-0.13}$	0.08	2.23	0.88	7.72	0.86
SNe Ia	0.29 $^{+0.19}_{-0.17}$ $^{+0.34}_{-0.21}$	-0.40	-5.41	1.03	553.81	0.96
CMB/BAO	0.29 $^{+0.03}_{-0.01}$ $^{+0.07}_{-0.03}$	-0.18	-2.17	1.08	1.24	0.25
f_{gas}	0.28 $^{+0.01}_{-0.01}$ $^{+0.04}_{-0.02}$	-0.03	-0.04	1.61	95.10	1.73
$H(z) + \text{SNe Ia}$	0.28 $^{+0.06}_{-0.07}$ $^{+0.12}_{-0.15}$	-0.85	-12.64	1.02	561.63	0.95
CMB/BAO + f_{gas} + $H(z)$ + SNe Ia	0.30 $^{+0.01}_{-0.01}$ $^{+0.02}_{-0.02}$	-0.25	-3.04	1.06	659.87	1.01

luminosity distance, $\mu_0 = 42.38 - 5 \log_{10} h$ and h stands for the Hubble constant H_0 in units of 100 km/s/Mpc. In our analysis, we use the latest Union2.1 sample which includes 580 SNe Ia over the redshift range $0.1 \leq z \leq 1.75$. In order to include the effect of systematic errors into our analyses, we follow [21] and estimated the best fit and error bars to the set of parameters \mathbf{P} by using a χ^2 statistics, with

$$\chi^2_{SNe} = \sum_{i=1}^{580} (\mu_i^{obs} - \mu_i^p)(\mathbf{C}^{-1})_{ij}(\mu_j^{obs} - \mu_j^p), \quad (11)$$

where \mathbf{C} is a 580×580 covariance matrix [21]. As for the analysis involving $H(z)$ data, we also marginalize over H_0 .

C. Results

In Fig. 2 (left) we show the first results of our statistical analyses. Contours corresponding to $\Delta\chi^2 = 2.3$ and $\Delta\chi^2 = 6.17$ in the $\Omega_{m0} \times c_2$ plane are shown for the χ^2 given above. From this analysis, we clearly see that neither the current $H(z)$ and SNe Ia measurements alone

nor the combination of them ($\chi^2 = \chi^2_{SNe} + \chi^2_H$) can place tight constraints on the values of the $f(R)$ parameter c_2 . We also observed that other cosmological observables seem to be unable to provide orthogonal contours to those shown in Fig. 2 (left), which in principle would lead to tighter bounds on c_2 from a joint analysis. Fig. 2 (right) shows the results obtained when two other sets of cosmological observations are included, namely: 7 estimates of the ratio of the CMB acoustic scale ℓ_A and the baryonic acoustic oscillation (BAO) peak, as given in Ref. [23], and the 57 measurements of the gas mass fraction in X-ray luminous galaxy clusters discussed in Ref. [24] (we refer the reader to Refs. [25] for more details on these cosmological tests). For this joint analysis, the best-fit values are $\Omega_{m0} = 0.30$ and $c_2 = -0.25$, with the reduced $\chi^2_r \equiv \chi^2_{min}/\nu \simeq 1.01$ (ν is defined as degrees of freedom), which is very close to the value found for the standard Λ CDM model, $\chi^2_r \simeq 1.001$. This result clearly shows that these scenarios are indistinguishable from each other at the background level, which coincides with the analysis performed in the metric formalism and discussed in Ref. [34] (although the expansion histories for both metric and Palatini formalisms are completely different). It is worth mentioning that in our analysis we consider both

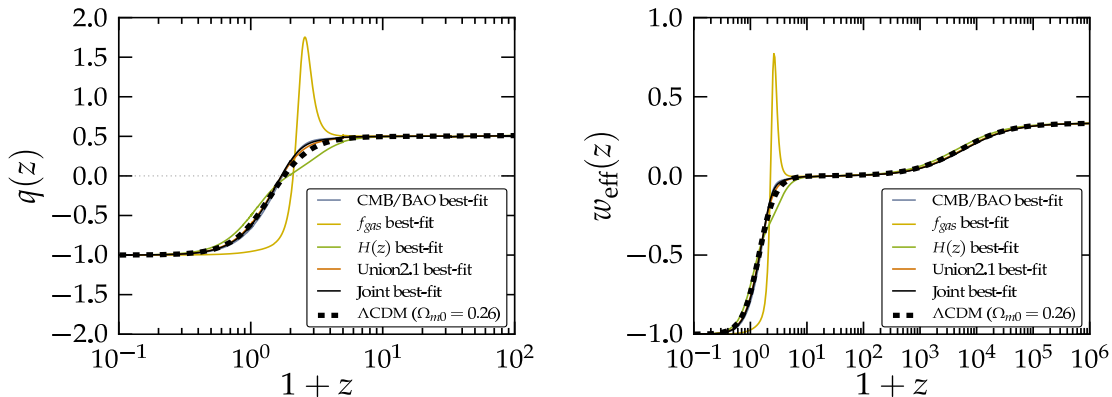


FIG. 3: Deceleration parameter (*left*) and effective equation of state (*right*) as a function of z for the best-fit values of Ω_{m0} and c_2 presented in Table I. The Λ CDM model is also shown for the sake of comparison.

positive and negative values of the $f(R)$ parameter c_2 , in agreement with the constraint (9). Note also that for $c_2 = 0$, the cosmological scenario discussed above reduces to the Einstein-de Sitter model, being ruled out by current data. We use the joint best-fit value discussed above, as well as the others shown in Table I, to discuss some cosmological consequences of this class of $f(R)$ cosmologies in the next section.

IV. COSMOLOGICAL CONSEQUENCES

Two important quantities directly related to the expansion rate of the Universe and its derivatives are the deceleration parameter

$$q(z) = \frac{(1+z)}{H(z)} H'(z) - 1, \quad (12a)$$

and the effective equation of state (EoS)

$$w_{eff} = -1 + \frac{2(1+z)}{3H} H'(z), \quad (12b)$$

where a prime denotes differentiation with respect to z and $H(z)$ is given by Eq. (4). For all sets of best-fit values displayed in Table I, we show $q(z)$ and $w_{eff}(z)$ as a function of the redshift in Fig. 3. Similarly to the analyses presented in Sec. III, we have included a component of radiation ($\Omega_{r0} = 5 \times 10^{-5}$) to plot these curves. As can be seen from Fig. 3 (left), for some combinations of parameters the cosmic evolution is well-behaved, with a past cosmic deceleration and a current accelerating phase starting at $z \simeq 1$, which seems to be in good agreement with current kinematic analyses (see, e.g., [33]).

For all best-fit combinations shown in Table I, Fig. 3 (right) shows that the Universe went through the last three phases of cosmological evolution, i.e., a radiation-dominated ($w_{eff} = 1/3$), a matter-dominated ($w_{eff} = 0$) and a late-time accelerating

phase ($w_{eff} < 0$), which is similar to what happens in the standard Λ CDM cosmology. From these results, it is also clear that the arguments of Ref. [10] about the behavior of w_{eff} in the metric approach (we refer the reader to [11] for a different conclusion) seems not to apply to the Palatini formalism, at least for the class of models discussed in this paper and the interval of parameters Ω_{m0} and c_2 given by our statistical analysis – the same also happens to power-law [12, 18] and exponential [19] $f(R)$ gravities in the Palatini formalism. Note also that, differently from exponential-type $f(R)$ models (see, e.g., [19]), we have not found solutions of transient cosmic acceleration in which the large-scale modification of gravity will drive the Universe to a new matter-dominated era in the future.

V. CONCLUSIONS

$f(R)$ -gravity provides an alternative way to explain the current cosmic acceleration with no need of invoking either the existence of an extra spatial dimension or an exotic component of dark energy. Among a number of $f(R)$ models discussed in the literature, the so-called Hu-Sawicki scenarios are designed to possess a chameleon mechanism that allows to evade solar system constraints. Although the cosmological scenario that arises from the metric formalism of this model has been shown to satisfy the conditions needed to produce a cosmologically viable expansion history, its Palatini version had not yet been investigated.

In this paper, we have discussed several cosmological consequences of this class of models in the Palatini formalism. We have performed consistency checks and tested the observational viability of these scenarios by using one of the latest SNe Ia data, the so-called Union2.1 sample, measurements of the expansion rate $H(z)$ at in-

termediary and high- z , estimates of the CMB/BAO ratio and current observations of the gas mass fraction in X-ray luminous galaxy clusters. We have found a good agreement between these observations and the theoretical predictions of the model, with the reduced $\chi^2_{min}/\nu \simeq 1$ for the analyses performed (see Table I). In what concerns the past cosmic evolution predicted by this class of models, we have shown that for a subsample of model parameters, the so-called Hu-Sawicki scenarios in the Palatini

approach are indistinguishable from the standard Λ CDM model, being able to produce the sequence of radiation-dominated, matter-dominated, and accelerating periods without need of dark energy.

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